ADVANCED ENGINEERING MATHEMATICS with Modeling Applications
Dedication

In memory of my parents
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Preface

This book springs from class notes, I developed for a course called Engineering Analysis which I have taught every other fall semester since 1983 at the University of Akron. The course is targeted to students who are beginning graduate study in engineering. The students enrolled are first- and second-year graduate students in the Department of Mechanical Engineering, although I have taught students from other engineering disciplines.

At the beginning of the first class, I tell my students that I have two objectives in teaching the class. The first is to prepare them for subsequent graduate courses in engineering by teaching mathematical methods that are used in major graduate classes. The second objective is to prepare students to read engineering literature, such as journals and monographs. Students engaged in thesis and dissertation research need a foundation in mathematical terminology and methods to understand previous work in their research area. The title of the course, Engineering Analysis, is vague, but it essentially means “Advanced Engineering Mathematics with Applications.” The course content and the content of this book is exactly that. Contrary to many engineering mathematics courses, the applications are emphasized as well as the physics behind the applications.

The applications are directed toward problems encountered in graduate engineering classes as well as in emerging areas of practice. An understanding of the modeling methods used to derive the mathematical equations as well as the underlying physics of the problem is usually essential to developing a method to solve the mathematical equations. For this reason, issues of modeling and scaling are discussed along with the analysis methods.

The motivation for the course and this book is to provide students and readers an experience of the marriage of engineering and applied mathematics. As in a marriage, both are equal and complement one another.

A unique feature of this book is the foundation laid for study. Books and courses on real analysis and functional analysis concentrate on theory rather than applications. Graduate engineering courses in areas, such as vibration, stress analysis, fluid mechanics, and heat transfer, concentrate on applications and use mathematical methods such as eigenvalue analysis and separation of variables as tools without much explanation of the mathematical theory underlying why they can be applied. An engineering student should understand the underlying mathematics that explains why these methods work,
when they can be applied, and what are their limitations, but does not need to understand how to prove every theorem. This book takes such a view.

An underlying foundation is developed using the language of vector spaces and linear algebra. Basic results are derived, such as the existence of energy inner products for self-adjoint operators, the eigenvector expansion theorem, and the Fredholm alternative, and applied to problems for discrete and continuous systems. Yes, the differences between finite and infinite dimensional spaces are addressed, especially the issues of convergence. For this study, a discussion of completeness of eigenvectors of self-adjoint operators and heuristic proofs of convergence with respect to energy norms is sufficient, and only a limited discussion of pointwise convergence of the trigonometric Fourier and Fourier-Bessel series is presented. Because the focus is on applied mathematics for beginning graduate students, topics such as continuous spectra of eigenvalues are omitted.

This book is different from other advanced engineering mathematics books in many ways, some of which are listed below:

- Applications are presented to provide motivation for the mathematics, whereas most existing books illustrate applications after developing the mathematics.
- Linear algebra is used to provide a foundation for analysis of discrete and continuous systems.
- Rigor is used in development of concepts and is used in proving theorems for which the proofs themselves are instructive.
- The view is taken that a general understanding of theory is necessary to develop applications.
- Applications from emerging technologies are presented.

Theorems and proofs are presented, but without the detail found in many mathematics books. It is not intended to have the development of the theory obscure its application to engineering problems. On the other hand, a full understanding of the solution is not possible unless the theory from which it is developed is understood.

Acknowledgement is due to students who have taken Engineering Analysis over the years. Their questions and suggestions helped refine the book. I gratefully acknowledge my wife, Seala Fletcher-Kelly, not just for her support, but for significant help in preparing the figures. I am grateful to B.J. Clark, formerly of Taylor & Francis, for his efforts in developing the project, as well as to Michael Slaughter and Jonathan Plant at CRC Press for their continued support during the project. I would also like to express appreciation for the work of Amber Donley, project coordinator, at CRC and Glenon Butler, project editor at Taylor & Francis and others at CRC and Taylor & Francis who aided in the publication of this work.

S. Graham Kelly